A Method of deducing the Formula for correcting the computed Time of an observed Occultation for Errors in the Elements adopted.

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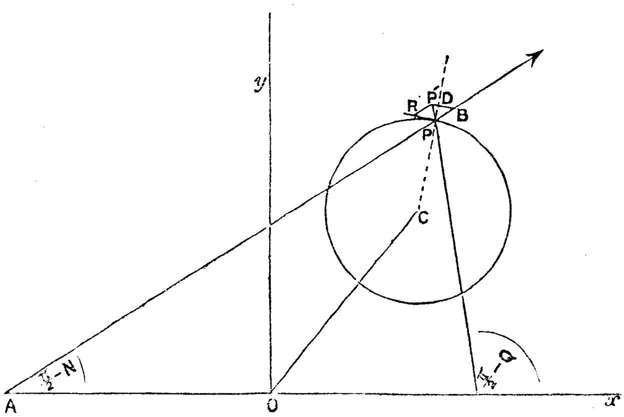
In Bessel's method of computing occultations the positions of the Moon and observer are referred to three axes at right angles to one another through the centre of the Earth: of which the axis of z is directed to the star (R.A. $= \alpha'$, Declination $= \delta'$), the axis of x to the point in the equator whose R.A. is $90° + \alpha'$, and the axis of y to the point in the declination-circle through the star whose distance from the equator is $90° + \delta'$. When the occultation takes place the projection of the place of observation on the plane of xy lies on the circumference of the projection of the Moon's disk. Let the figure represent the projection on the



plane xy of the Moon's disk at the time T, and let P be the projection of a place at which the occultation is taking place. Let P$'$ be an adjacent point in the plane: through P and P$'$ draw lines parallel to the projected direction of motion of the Moon's centre at the time T, and draw PR touching the circle at P. Then if PP$'$ be small, P$'$R is approximately the distance that the Moon's projection must pass through before the point P$'$ is on the circumference. Draw P$'$B parallel to the tangent at P, cutting CP produced in D. Let PP$'$ $= d$, and the angle PP$'$ makes with the axis of $x = 90° - $Q.

With Chauvenet's notation the angle PA$x = 90° - $N, angle CPB $= \psi$; and the Moon's relative velocity in its projected path is n,—the unit of length being the equatoreal radius of the Earth and of time one mean hour.

Now, from the figure—

$$PP' \cos DPP = PD = BP \cos BPD,$$

$$\text{that is} \quad PP' \cos (BPP' - BPD) = P'R \cos BPD;$$

$$\text{or} \quad d \cos \{N - Q - (180^\circ - \psi)\} = P'R \cos (180^\circ - \psi);$$

$$\therefore P'R = \frac{d \cos (N - Q + \psi)}{\cos \psi};$$

and the time of Moon's projection describing $P'R = \frac{P'R}{n}$. If P′ be the projection of a point on the Earth's surface not far distant from P, the time at which the occultation takes place at this second station will be approximately $T \pm \frac{P'R}{n}$. A paper on this case by the Rev. T. Chevallier, B.D., will be found in Vol. XIX. of the *Memoirs*. Now consider the case in which the projection of the Moon's centre undergoes slight changes of position owing to slight changes in the elements. These changes may for convenience be attributed to the point P with opposite signs. A slight change common to both Moon and star has scarcely any effect, so we need only consider relative changes.

(i.) If the relative increase of R.A. of the Moon and star be $\Delta (\alpha - \alpha')$, then, π denoting the Moon's parallax, the Moon's centre will be moved through an arc equal to $\frac{\Delta (\alpha - \alpha') \cos \delta}{\sin \pi} \times$ circular measure of 1″, perpendicular to the plane zy; therefore, in this case, PP′ is equal to $\frac{\Delta (\alpha - \alpha') \cos \delta}{\pi}$ and parallel to xO, that is $Q = 90^\circ$.

$$\therefore P'R = -\frac{\Delta (\alpha - \alpha') \cos \delta}{\pi} \cdot \frac{\sin (N + \psi)}{\cos \psi}.$$

(ii.) Let $\Delta (\delta - \delta')$ be the relative increase of declination. Here PP′ is equal to $-\frac{\Delta (\delta - \delta')}{\pi}$ and parallel to yO, that is $Q = 0$.

$$\therefore P'R = -\frac{\Delta (\delta - \delta')}{\pi} \cdot \frac{\cos (N + \psi)}{\cos \psi}.$$

(iii.) Let $\Delta\kappa$ be the increase of the Moon's radius. Then P′ is on the line APB, that is $Q = N$.

$$\therefore P'R = PP' = \frac{-\Delta\kappa}{\cos (180 - \psi)} = \frac{\Delta\kappa}{\cos \psi}.$$

(iv.) Let $\Delta\pi$ be the increase in the Moon's parallax. Put $OC = R$ and angle $COx = 90^\circ - \theta$: then from this change C moves towards O a distance ΔR.

Now $R = \frac{a}{\sin \pi}$ where a is independent of π.

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$$\therefore \frac{\Delta R}{R} = -\frac{a \cos \pi}{\sin^2 \pi} \cdot \frac{\sin \pi}{a} \Delta\pi \times \text{circ. meas. of } 1'' = -\frac{\cos \pi}{\pi} \cdot \Delta\pi,$$

$$\therefore PP' = -\Delta R = \frac{R \cos \pi}{\pi} \cdot \Delta\pi, \text{ and } Q = \theta,$$

$$\therefore P'R = \frac{R \cos \pi . \Delta\pi}{\pi} \cdot \frac{\cos (N+\psi) \cos \theta + \sin (N+\psi) \sin \theta}{\cos \psi}$$

$$= \frac{\cos \pi . \Delta\pi}{\pi} \cdot \frac{y \cos (N+\psi) + x \sin (N+\psi)}{\cos \psi},$$

where x and y are the co-ordinates of the Moon's centre at the time T.

Since these small changes are independent of one another, their combined effect will be equal to their sum.

$$\therefore \left.\begin{array}{c}\text{total correction to the}\\ \text{time T in seconds}\end{array}\right\} = \frac{\text{sum of the several values of } P'R}{n} \times h,$$

where h is the number of seconds in the unit of time; that is

$$\text{correction} = -\frac{h}{n\pi \cos \psi} [\sin (N+\psi) \cos \delta . \Delta (\alpha - \alpha') + \cos (N+\psi) \Delta (\delta - \delta')$$
$$+ \pi . \Delta\kappa - \{x \sin (N+\psi) + y \cos (N+\psi)\} \cos \pi . \Delta\pi],$$

which is the usual formula, omitting the terms depending on error of eccentricity of the meridian.

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On the Effect of Wear in the Micrometer-Screws of the Greenwich Transit-Circle. By W. H. M. Christie, Esq.

It has been the practice of observers to examine carefully the accuracy of micrometer-screws on receiving them from the maker's hands; but no one, so far as I am aware, has considered it necessary to make any observation afterwards, though the effect of wear, when observations are made frequently, soon becomes sensible. Under these circumstances, it may be desirable to give an account of a case in which considerable errors have been caused by continued use during a period of more than twenty years.

The wear in the micrometer-screws of the Greenwich Transit-Circle first showed itself by a discordance between the Zenith-points deduced from the Nadir observation and from Stars respectively; though the cause was not suspected till the beginning of 1875, when a change in the position of the division in the field of view of the micrometer-microscopes was found to be accompanied by a change in the discordance from $+0''\cdot7$ to $-0''\cdot6$. This discordance first became sensible in the year 1868; but the error in the